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TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 1143
COURSE	: DISCRETE MATHEMATICS
SEMESTER/SESSION	: 1-2022/2023
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **5** questions in SECTION A, **3** questions in SECTION B and **2** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 7 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

a) Determine the truth or falsity of each of the following statements.

i. $-5 > -1$ or $2+4=6$ (1 mark)

ii. $2+2=3$ and $5>1$ (1 mark)

b) Assuming that p and r are FALSE and that q is TRUE, find the truth value of each proposition.

i. $[q \wedge (p \rightarrow r)] \rightarrow q$ (3 marks)

ii. $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$ (3 marks)

QUESTION 2

a) Let $a=88$ and $b=126$. Find the greatest common divisor (gcd) and least common multiple (lcm) for both integers a and b by using prime factorization. (4 marks)

b) Convert $(7345321)_8$ to hexadecimal expansion. (4 marks)

QUESTION 3

a) Given that $A=\begin{bmatrix} 10 & -3 & 4 \\ -6 & 0 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$, and $C=\begin{bmatrix} -1 & 3 & -7 \\ 1 & 5 & 2 \end{bmatrix}$. Compute $(A+BC)$. (4 marks)

b) Given that $A=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ are Boolean matrix. Compute $A \vee B$, $A \wedge B$ and $A \odot B$. (3 marks)

QUESTION 4

a) How many edges are there in a graph with 12 vertices each of degree three?

(2 marks)

b) Draw a graph $G=(V,E)$, where $V=\{a,b,c,d,e\}$, $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ and $e_1=e_5=\{a,c\}$, $e_2=\{a,d\}$, $e_3=\{e,c\}$, $e_4=\{b,c\}$ and $e_6=\{e,d\}$.

(5 marks)

c) Determine whether or not the graph in Figure 1 is bipartite. Give the bipartition sets or explain why the graph is not bipartite.

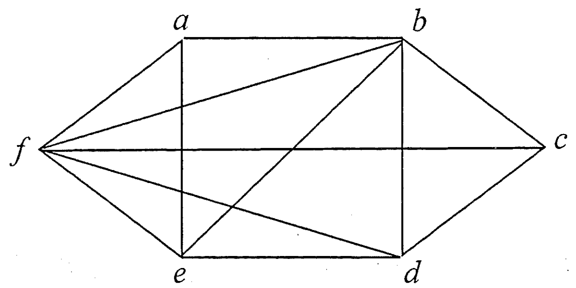

Figure 1

(2 marks)

d) Draw $K_{2,6}$. (2 marks)

QUESTION 5

c) Let $A=\{0,1,2,3,4\}$, $B=\{0,1,2,3\}$ and define relations $R=\{(a,b)\mid \operatorname{lcm}(a,b)=2\}$ and $S=\{(a,b)\mid a+b=4\}$ from A to B. State explicitly which ordered pairs are in

i. $A\times B$ (1 mark)

ii. R (2 marks)

iii. S (2 marks)

a) Given $A=\{1,2,3,4\}$, its relation is $R=\{(1,2),(1,3),(1,4),(2,1),(2,3),(3,1)\}$.

i. Draw a digraph for the relation R. (2 marks)

ii. Determine whether or not the binary relations R defined on the sets A are reflexive, symmetric or transitive. Explain why or why not?

(3 marks)

b) Let $A=\{2,4,6\}$. List the ordered pairs in a relation on A which is:

i. not reflexive, not symmetric and not transitive. (2 marks)

ii. reflexive, but neither symmetric nor transitive. (2 marks)

iii. symmetric and transitive, but not reflexive. (2 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Prove by mathematical induction that:

a) $1+2\cdot 2+3\cdot 2^2+\cdots+(n+1)2^n=n\left(2^{n+1}\right)$, for $n\geq 1$ (8 marks)

b) $1^3+2^3+\cdots+n^3=\dfrac{n^2(n+1)^2}{4}$, for $n\geq 1$ (8 marks)

QUESTION 2

Let $X=\{x\in\mathbb{Z}\mid 1<x<7\}$, $Y=\{y\in\mathbb{R}\mid y^2-5y+6=0\}$ and $Z=\{z\in\mathbb{N}\mid z \text{ is a perfect square and } z\leq 20\}$.

a) List the elements in each of these sets. (3 marks)

b) Find $X\cup Y\cup Z$, $X\cap Z$, $X\setminus Y$ and $X\setminus Z$. (4 marks)

QUESTION 3

How many bit strings of length 10 contain:

a) exactly four 1s? (1 mark)

b) at most four 1s? (3 marks)

c) an equal number of 0s and 1s? (3 marks)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Fifteen people on a softball team show up for a game.

a) How many ways are there to choose 10 players to take the field? (2 marks)

b) How many ways are there to assign the 10 positions by selecting players from the 15 people who show up? (2 marks)

c) Of the 15 people who show up, 4 are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman? (4 marks)

QUESTION 2

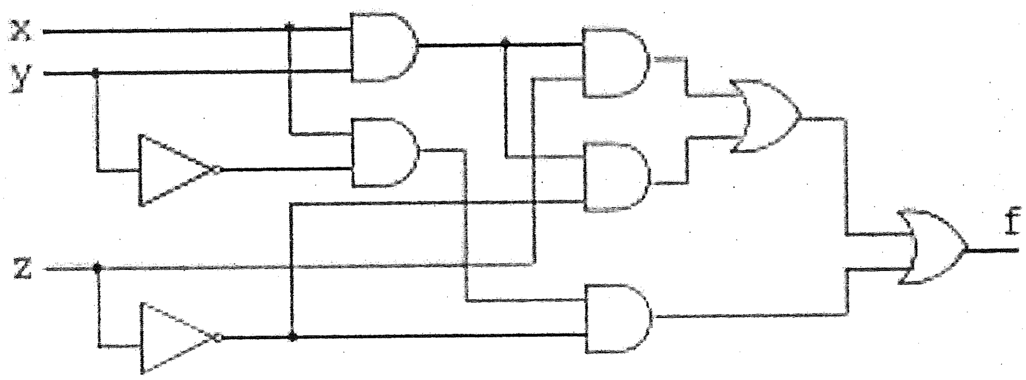

Figure 2

a) Find the expression for the Boolean function in Figure 2. (2 marks)

b) Construct the truth table for the output. (3 marks)

c) Simplified the Boolean function and sketch the circuit for the simplified function. (7 marks)

...................................END OF QUESTIONS...

FORMULA

Identity	*Name*
$\bar{\bar{x}} = x$	Law of the double complement
$x + x = x$ $x \cdot x = x$	Idempotent laws
$x + 0 = x$ $x \cdot 1 = x$	Identity laws
$x + 1 = 1$ $x \cdot 0 = 0$	Domination laws
$x + y = y + x$ $x \cdot y = y \cdot x$	Commutative laws
$x + (y + z) = (x + y) + z$ $x \cdot (y \cdot z) = (x \cdot y) \cdot z$	Associative laws
$x + (y \cdot z) = (x + y) \cdot (x + z)$ $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$	Distributive laws
$\overline{x \cdot y} = \bar{x} + \bar{y}$ $\overline{x + y} = \bar{x} \cdot \bar{y}$	De Morgan's laws
$x + (x \cdot y) = x$ $x \cdot (x + y) = x$	Absorption laws
$x + \bar{x} = 1$	Unit property
$x \cdot \bar{x} = 0$	Zero property